# Transition phenomena in oscillating boundary-layer flows

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The transition Reynolds number and the turbulent Reynolds number induced by a sinusoidally fluctuating free stream have been determined experimentally. The oscillating flow was produced in a closed-circuit wind tunnel by means of a rotating shutter valve which had a range of frequencies from 4 to 125 c/s corresponding to a range of the dimensionless frequency parameter, $\omega\nu/U_\infty^2$, of $2{\cdot}29\times10^{-6}$ to $4{\cdot}49\times10^{-5}$. The dimensionless amplitude parameter, $\Delta U/U_\infty$, could be adjusted by means of shutter blades of various widths from a value of 0·0775 to 0·667.

Flows in both the free stream and the boundary layer were monitored simultaneously by means of two transistorized constant-temperature hot-wire anemometers. The transition Reynolds number, the turbulent Reynolds number and turbulent intermittency factor, $\gamma$, were determined from the velocity-time traces recorded on a dual-channel oscilloscope.

It was found that the transition Reynolds number depends only on the amplitude of the oscillations and that the dimensionless transition length is a function only of the frequency. The time-space distribution of turbulent bursts in the transition region indicates that the location as well as duration of bursts is quite regular and closely tied to the fluctuations of free-stream velocity, confirming the analysis of Greenspan & Benney.

## Introduction

The influence of free-stream turbulence and streamwise pressure gradients on transition from laminar to turbulent flow in boundary layers has been the subject of numerous investigations, for example, Schlichting (1962), Schubauer & Skramstad (1943) and Taylor (1938). On the other hand, large oscillations of the free-stream velocity have not been considered in this context. The present study is concerned with transition in Blasius-type boundary layers produced by a free stream having an oscillating component of velocity.

## Transition in Blasius flow

Steady viscous flow over a semi-infinite flat plate without streamwise pressure gradient, termed Blasius flow, represents the simplest type of a boundary-layer flow and the one about which the greatest fund of knowledge has been accumulated. A reasonably complete qualitative picture of transition for Blasius flow has already been assembled.

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Proceeding downstream with the flow from the leading edge of the plate a sequence of events are observed which ultimately lead from an initially laminar boundary-layer flow to a fully established turbulent flow. As will be shown, a complete description of this process requires at least three characteristic Reynolds numbers.

A hot-wire anemometer placed in the laminar boundary layer near the leading edge reveals the existence of random disturbances corresponding in magnitude to the turbulence level of the free stream. When the probe is moved further downstream past a point, referred to in the stability analysis of Tollmien–Schlichting as the critical Reynolds number ($N_{Re,c}$), a selective amplification of these disturbances takes place. This phenomenon cannot be readily observed unless the turbulence level of the free stream is exceedingly low, as was the case in the experiments of Schubauer & Skramstad which confirmed the Tollmien–Schlichting neutral stability curve. The selective amplification of the random disturbances results in a regular two-dimensional pattern of sinusoid-like disturbances, referred to as Tollmien–Schlichting waves, which increase in amplitude slowly with increasing Reynolds number until interrupted by short random bursts of large-scale disturbances referred to as turbulence. Typically, the appearance of turbulent bursts occurs at a Reynolds number, referred to as the transition Reynolds number ($N_{Re,t}$), which exceeds the critical Reynolds number by an order of magnitude. The régime between the transition Reynolds number and the turbulent Reynolds number is characterized by an intermittency factor, $\gamma$, defined as the fraction of the time during which the flow of a given position remains turbulent.

It has been observed that this breakdown of the Tollmien-Schlichting waves into turbulence is not a sudden spatial phenomenon, but a sequence of random three-dimensional temporal bursts of turbulence first appearing at the transition Reynolds number and becoming more frequent with increasing Reynolds number until at some third critical Reynolds number, termed the turbulent Reynolds number, ($N_{Re,T}$), the flow becomes wholly turbulent. Thus, there are three characteristic Reynolds numbers necessary to complete description of the transition phenomenon.

An understanding of the development of turbulent fluctuations in the region between the transition and turbulent Reynolds numbers requires an analysis of the stability of the Tollmien–Schlichting flow to finite disturbances. This problem has been approached by Stuart (1960) and Watson (1960) and more recently by Greenspan & Benney (1963). Based on on a mapping of the flow in the Tollmien–Schlichting region by Kovasznay, Komoda & Vasudeva (1962), and Klebanoff, Tidstrom & Sargent (1962), Greenspan & Benney have formulated an approach to the question of the linear stability of the Tollmien–Schlichting flow to large disturbances. This was accomplished by introducing a number of approximations of the Blasius velocity profile modified by the Tollmien–Schlichting flow and computing the turbulent energy amplification rate. Although no general solution can be obtained in this manner, a qualitative understanding of the results was developed by approximate analysis of the equations from which order-of-magnitude results can be calculated. The results indicate that the intensity of shear is the critical parameter in control of the energy amplification and the shear-layer thickness emerges as the parameter controlling the wave-number of frequency sensitivity. The former of these two results was correctly anticipated by Liepmann (1945).

The Greenspan–Benney analysis confirms the notion that instability of the Tollmien–Schlichting flow is the mechanism responsible for the sudden amplification of high-frequency disturbances and breakdown into turbulence. Moreover, it leads to a picture of the 'continuous creation of local instabilities generating

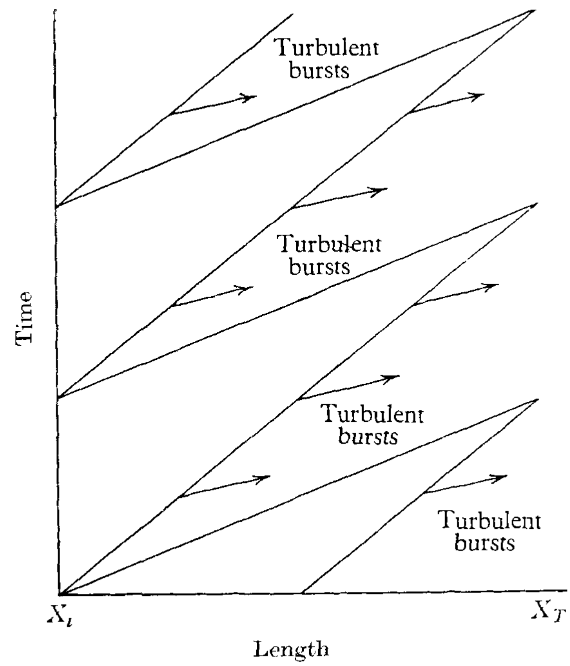


FIGURE 1. Theoretical time-space distribution of turbulent bursts in the transition region.

turbulent spots at favourable positions and times corresponding to the most intense shear layer'. Figure 1 is a time-space map of such instabilities taken from Greenspan & Benney. It should be noted that both the assumed disturbances and the resulting breakdown pattern were two-dimensional while actual steady-flow breakdown patterns have been observed to be three-dimensional in character.

In order to determine to what extent the phenomena associated with transition in steady flows, described above, are related to transition in an oscillating stream, the characteristics of boundary layers in oscillating streams must be considered.

It has been shown by Hill (1958), Karlsson (1958) and others that the mean value of the local velocity is not affected by the superposition of an oscillating component in either laminar or turbulent boundary layers. The mean flow then remains a solution of the Navier–Stokes equations and may be subtracted from the linearized equations of motion. Thus, the resulting disturbance equations are identical to those obtained for steady flow and the Tollmien–Schlichting theory should be applicable to large amplitude oscillating flows. When the amplitude of the oscillating component of the free-stream velocity is equal to or greater than typical turbulence amplitudes, one may expect to find agreement between the transition process in the oscillating boundary layer and the description given by Greenspan & Benney for the steady case since spanwise variations are likely to be suppressed in the oscillating flow due to the two-dimensional character of the oscillations.

## Experimental apparatus

The present work was carried out in a low-speed closed-circuit wind tunnel having a 2 ft.-square test section and a velocity range of 10–100 ft./sec. Free-stream turbulence intensities, $(\overline{U}/U_\infty)$, from 0·585 to 0·667 were measured in this velocity range. Oscillations in the free-stream velocity were introduced by means of a rotating shutter valve (figure 2, plate 1), located between the test section and the diffuser.

The shutter valve consists of a frame supporting four shafts equally spaced across the section height. Each shaft is provided with a slot into which flat blades of various widths may be introduced, forming a set of four butterfly valves spanning the cross-section. The range of widths of blades employed allowed perturbations in the range of 8–92 % of the free-stream velocity to be introduced. The valve was driven by a variable-speed electric motor which provided a range of frequencies from 4 to 125 c/s.

The top of the test section was fitted along its centre line with a slot covered by a sliding strip of spring steel for its entire length with the exception of two $\frac{1}{4}$-in. holes through which hot-wire anemometer probes were inserted. A traversing mechanism seen in figure 2 allowed these probes to be positioned axially with a precision of 0·010 in. One of the probes was attached to a micrometer mechanism, having a range of 1 in., which was used for traversing the boundary layer. The second probe, used for measurements in the free stream, was located in the same transverse plane as the traversing probe.

The flat plate model, fabricated of $\frac{3}{4}$-in. aluminium plate, $23\frac{3}{4}$ in. wide and 36 in. long is shown installed in the test section in figure 2. The leading edge was milled to a 10° wedge with a 0·010 in. tip radius. The plate was hand lapped to a 20 $\mu$in. finish after all scratches and nicks were filled with an epoxy compound.

The forward stagnation point was positioned approximately 0·025 in. from the leading edge on top of the flat plate with the aid of an adjustable flap, seen in figure 2, plate 1, located on the upper wall directly over the trailing edge of the plate. Static-pressure surveys along the centre-line of the plate indicated that the flap had a negligible effect on the pressure distribution over the first 24 in. of the plate and that the pressure gradient, represented there by a Euler number of 0·0292, was approximately zero.

The constant-temperature hot-wire anemometers used for the velocity measurements are based on the design of L. S. G. Kovasznay and consist of transistorized, d.c.-coupled circuits having an estimated frequency response of 17 kc/s. A self-contained transistorized analogue computer serves to invert King's Law and yields an output which is linear in velocity. The d.c. coupling permits the output to be separated into steady and oscillating components which is otherwise not possible. The wire filaments employed were 0·080 in. long and were prepared from 0·00015 in. tungsten wire.

Output from the two hot-wire channels was displayed on a two-channel oscilloscope and recorded photographically. The d.c. and a.c. components of the hot-wire signals were measured with a vacuum-tube voltmeter and a Ballantine True R.M.S. Voltmeter, respectively. Harmonic analyses of the velocity-time traces were determined with a vibration analyser equipped with a selective band-pass filter.

Calibration of the hot-wire anemometers was carried out *in situ* using a conventioned Prandtl probe as a standard.

## Characteristics of the flow

In order to insure that the experimental apparatus and instrumentation would produce results consistent with previous work, the transition and turbulent Reynolds numbers were measured in a steady flow at a free-stream turbulence

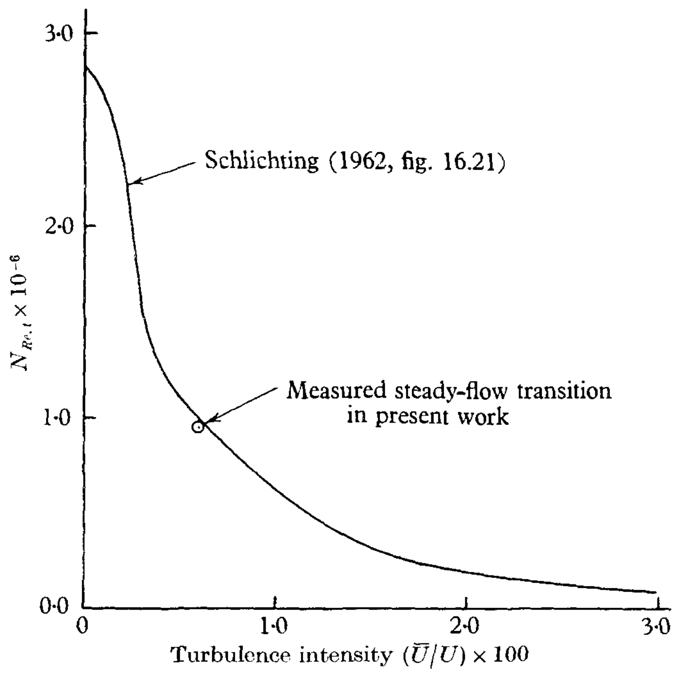


FIGURE 3. Effect of free-stream turbulence on steady flow transition Reynolds number.

intensity of 0·625 % and found to be $9{\cdot}54 \times 10^5$ and $1{\cdot}276 \times 10^6$, respectively. The former of these two points is shown with the data of Schubauer, Dryden and Hall and Hislop reported by Schlichting (1962) in figure 3.

With the shutter valve in operation laminar and turbulent velocity profiles were determined under flow conditions similar to those of Hill and Karlsson, respectively.

In the laminar region the mean velocity profile was found to agree closely with the Blasius profile, as had been observed by Hill. Distribution of the local oscillation amplitude parameter, $\Delta u/U_\infty$, confirmed the theoretical results of Nickerson (1957) and the measurements of Hill in the laminar boundary layer for the low-frequency régime. In the high-frequency régime, it was found that the analytical treatment of Lin (1956) adequately described the flow.

In the turbulent boundary layer the distribution of the local amplitude parameter, $\Delta u/U_\infty$, was found to agree with the measurements of Karlsson who employed similar means for introducing oscillations and the same type of instrumentation.

The boundary-layer oscillations were observed to be always in phase with the fluctuations in the free stream; however, these observations were limited to distances from the wall in excess of 0·010 in. The study of Lin, applicable to large-amplitude oscillations, predicts differences in phase at points closer than could be conveniently surveyed. On the other hand, phase differences have been reported by Hill for large distances from the wall. However, from the description of the apparatus used by Hill, it appears that these apparent phase differences may have been the result of distortions due to frequency-compensation problems associated with the constant-current hot-wire anemometer employed in that case. Attention has been called to difficulties of this type by Hinze (1959).

The accuracy with which the rotary valve produced sinusoidal oscillations was also considered. Harmonic analyses during two runs revealed that for both cases four terms of the Fourier series adequately represented the wave form within 1 %. These results are similar to those of Feiler & Yeager (1962), who produced an oscillating flow by means of a rotating siren-like valve located upstream of the test section.

## The transition Reynolds number in oscillating flow

Transition Reynolds number, $N_{Re,t}$, have been determined in oscillating flow for a range of dimensionless amplitude, $N_A = \Delta U/U_\infty$, from 0·0775 to 0·667 in which range the transition Reynolds number appears to be independent of the oscillation frequency. The flow conditions are summarized for each of the runs in table 1 and the results are presented in figure 4 and table 2. In addition to these results, data by Hall & Hislop (1938) concerning the effect of free-stream turbulence intensity on transition are included in the figure and the critical Reynolds number of the Tollmien–Schlichting theory is also shown. The latter is approached by the transition Reynolds number as $N_A$ approaches unity. i.e. the amplitude of the velocity fluctuation approaches the mean of the free-stream velocity. It is apparent from the experimental results that the transition Reynolds number is independent of the frequency of the oscillations. This seems to be consistent with the conclusion reached by Liepman (1945) that in steady Blasius flow the transition Reynolds number is determined by the free-stream turbulence level.

## The turbulent Reynolds number

For each of the experimental runs in which complete transition was realized on the flat plate the turbulent Reynolds number, $N_{Re,T}$, is presented in table 2. The corresponding dimensionless transition length is shown in figure 5 as a function of the frequency parameter $N_F = \omega\nu/U_\infty^2$. It is apparent from this figure that the transition length is dependent on the frequency of the oscillations and is independent of their amplitude, as predicted by Greenspan & Benney. For values of the frequency parameter, $N_F$, of the order of $5 \times 10^{-3}$ the transition length approaches zero.

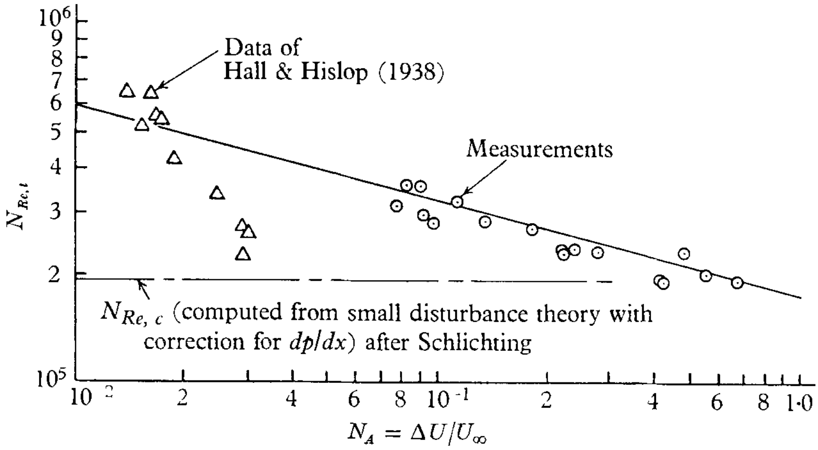


FIGURE 4. Effect of the amplitude parameter, $\Delta U/U_\infty$, on the transition Reynolds number.

| Run | $U_\infty$ (ft./sec) | $N_A = \Delta U/U_\infty$ | $\omega$ (c/s) | $N_F = \omega\nu/U_\infty^2 \times 10^6$ |
|---|---|---|---|---|
| 1 | 103·2 | 0·0775 | 20·0 | 2·29 |
| 2 | 101·1 | 0·0830 | 7·58 | 0·845 |
| 3 | 103·2 | 0·115 | 4·08 | 0·455 |
| 4 | 103·6 | 0·132 | 12·5 | 1·354 |
| 5 | 102·0 | 0·0910 | 39·7 | 4·58 |
| 6 | 61·3 | 0·667 | 11·38 | 3·58 |
| 7 | 56·6 | 0·487 | 4·16 | 1·51 |
| 8 | 63·0 | 0·429 | 28·6 | 8·65 |
| 9 | 62·7 | 0·421 | 22·7 | 6·86 |
| 10 | 87·1 | 0·186 | 4·16 | 0·651 |
| 11 | 77·0 | 0·222 | 15·15 | 3·10 |
| 12 | 75·8 | 0·223 | 27·8 | 5·85 |
| 13 | 74·8 | 0·280 | 45·4 | 9·63 |
| 14 | 105·7 | 0·115 | 4·08 | 0·427 |
| 15 | 102·9 | 0·136 | 15·9 | 1·79 |
| 16 | 107·6 | 0·0925 | 122 | 13·69 |
| 17 | 85·9 | 0·240 | 125 | 26·9 |
| 18 | 74·5 | 0·560 | 119 | 44·9 |

TABLE 1. Experimental operating conditions.

## Distribution of turbulent intermittency in the transition zone

The distribution of the intermittency factor, $\gamma$, in the transition region was determined from the oscilloscope traces and is shown for a typical run in figure 6. The corresponding sequence of oscilloscope traces is presented in figure 7. The striking regularity with which a turbulent burst appears in every cycle and

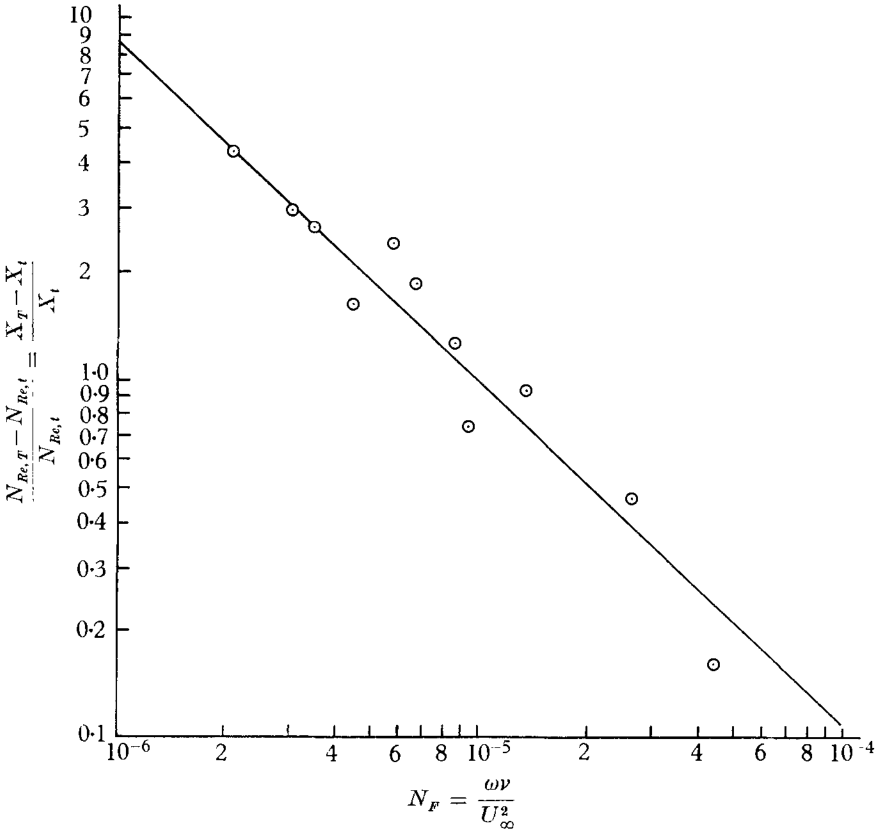


FIGURE 5. Effect of the frequency parameter, $\omega\nu/U_\infty^2$, on the transition length.

| Run | $N_A = \Delta U/U_\infty$ | $N_{Re,t}$ $\times 10^{-5}$ | $N_{Re,T}$ $\times 10^{-5}$ | $N_F = \omega\nu/U_\infty^2$ $\times 10^6$ | $N_L = \frac{N_{Re,T}-N_{Re,t}}{N_{Re,t}}$ |
|---|---|---|---|---|---|
| 1 | 0·0775 | 3·10 | 16·4 | 2·29 | 4·29 |
| 2 | 0·0830 | 3·58 | — | 0·845 | — |
| 3 | 0·115 | 3·81 | — | 0·455 | — |
| 4 | 0·132 | 4·20 | — | 1·354 | — |
| 5 | 0·0910 | 3·56 | 9·35 | 4·58 | 1·62 |
| 6 | 0·667 | 1·90 | 7·07 | 3·58 | 2·72 |
| 7 | 0·487 | 2·29 | — | 1·51 | — |
| 8 | 0·429 | 1·92 | 4·40 | 8·65 | 1·29 |
| 9 | 0·421 | 1·93 | 5·54 | 6·86 | 1·87 |
| 10 | 0·186 | 2·68 | — | 0·651 | — |
| 11 | 0·222 | 2·32 | 9·30 | 3·10 | 3·00 |
| 12 | 0·223 | 2·29 | 7·85 | 5·85 | 2·43 |
| 13 | 0·280 | 2·31 | 3·96 | 9·63 | 0·747 |
| 14 | 0·115 | 3·23 | — | 0·427 | — |
| 15 | 0·136 | 2·80 | — | 1·79 | — |
| 16 | 0·0925 | 2·95 | 5·72 | 13·69 | 0·940 |
| 17 | 0·240 | 2·29 | 3·38 | 26·9 | 0·476 |
| 18 | 0·560 | 2·00 | 2·33 | 44·9 | 0·165 |

TABLE 2. Transition and turbulent Reynolds numbers.

the similarity between the resulting picture shown in figure 6 and the qualitative results shown by Greenspan & Benney, figure 1, indicates the anticipated close relationship between transition processes in oscillating flows and those in steady flows subjected to two-dimensional, periodic free-stream disturbances.

Greenspan & Benney have predicted that such disturbances would remove the random spanwise variation of the intermittency factor observed by Schubauer & Skramstad in steady-flow transition. This effect seems to materialize as shown by spot checks of $\gamma$ made at a point located at a spanwise distance of 2 in. from the boundary-layer probe.

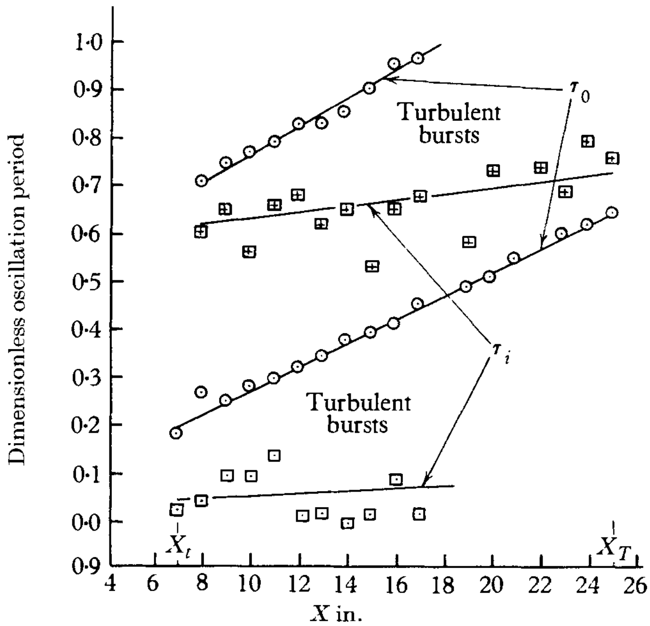


FIGURE 6. Measured time-space distribution of turbulent bursts in the transition region. Run 8: $N_A = 0{\cdot}667$, $N_F = 8{\cdot}65 \times 10^{-6}$.

The randomness in the streamwise distribution of the intermittency factor, $\gamma$, characteristic of steady-flow transition was not observed in the present work. This is traceable to the fact that an oscillating free-stream velocity component strong enough to eliminate the spanwise variation must do so by influencing the time-space variation of peak shear in the boundary layer. Since the peak value of shear has been shown to be the critical factor in transition, it must follow that if the oscillating free stream removes the spanwise randomness it must also remove the streamwise randomness. The time-space distribution of turbulent bursts predicted by Greenspan & Benney, figure 1, imply a linear streamwise variation in the intermittency factor. The measured distribution, figure 6, confirms this.

Variation with vertical distance in the values of the time corresponding to initiation of a turbulent burst, $\tau_i$, and the time corresponding to cessation of the turbulent burst, $\tau_0$, have been determined within the boundary layer at various distances from the wall at a single streamwise location. The variation across the boundary layer in $\tau_i$ was found to be 5·5 % and the variation in $\tau_0$ 6·0 %. The values of $\tau_i$ and $\tau_0$ appeared to be scattered randomly and no trend with change in vertical distance was detected.

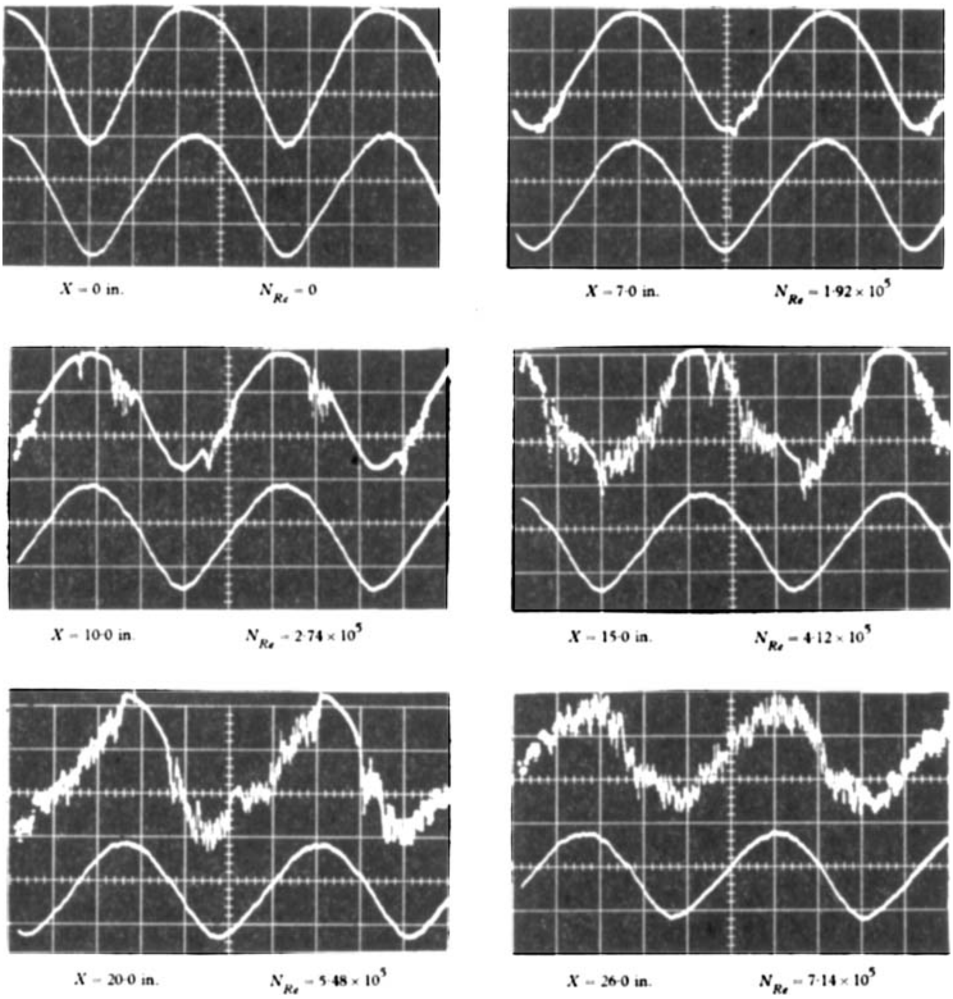


FIGURE 7. Oscilloscope traces illustrating the growth of the intermittency factor, $\gamma$, with increasing Reynolds number in the transition region. Upper trace: boundary-layer record. Lower trace: free-stream record.

## Conclusions

From the results the following conclusions may be drawn:

1. The transition Reynolds number, $N_{Re,t}$, is influenced only by the amplitude of the free-stream oscillation, as is shown in figure 4, confirming the prediction of Liepmann.

2. The dimensionless transition length is influenced only by the frequency of the free-stream oscillation, as shown in figure 5, confirming the prediction of Greenspan & Benney.

3. The time-space distribution of the intermittency function, $\gamma$, shown in figure 6, confirms the qualitative theoretical prediction of Greenspan & Benney shown in figure 1.

4. In oscillating flow the time-space distribution of the intermittency function, $\gamma$, is not random as it is in the case of steady flow, since the oscillating component of velocity rigorously influences the time distribution of shear which leads to breakdown.

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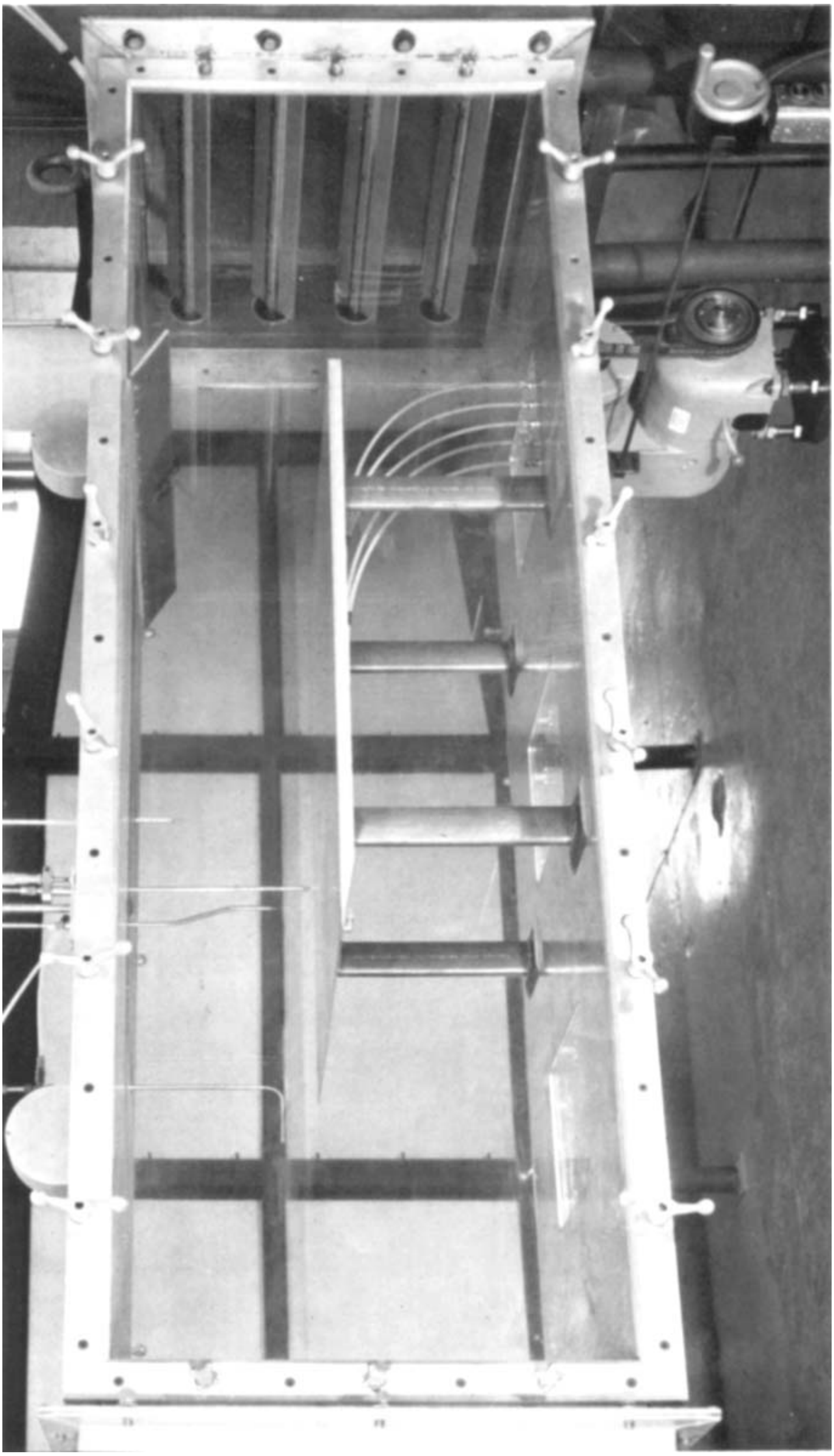

FIGURE 2. View of the experimental apparatus.